## Special Focus: <br> Inference

## Inferential Problems for Practice

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The following problems are provided for practice in writing answers to "typical" inference problems such as might appear on the free-response section of the AP Statistics Exam.

You will notice that "the" answers are not provided-you will have to engage in some discussion with your colleagues to arrive at acceptable, statistically informative, and reasonable methodology, communication, and "correctness."

## Inference for a Single Proportion

A recent study investigated the effects of a campaign to increase automobile drivers' awareness of designated bus lanes in a large midwestern city. Investigators observed a random sample of drivers over the course of the morning rush hour one day and found 92 out of 114 drivers were not "poaching" in the bus lanes.
(a) Compute a point estimate of the true proportion of all automobile drivers who stay out of the bus lanes.
(b) Construct and interpret a 95 percent confidence interval for $\pi$, the true proportion of all drivers who stay out of the bus lanes.
(c) Before the campaign it was found that 50 percent of drivers were not poaching in the bus lanes. Based on your responses in (a) and (b), is there evidence that the campaign has increased the automobile drivers' avoidance of bus lanes?

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## Inference for Two Proportions

When biologists study nesting birds, it is necessary to check the nests periodically to see if the young birds have hatched, are still alive, and so forth. In order to check the nest, the parents must be flushed from the nest, and some biologists are concerned that this human intervention may increase the likelihood of nest failure during the nesting season. In a recent study of nesting mourning doves, nests were randomly assigned to be "disturbed" or left alone for the nesting season. The nesting success of the two groups is presented below:

| Mourning Dove Nests |  |  |
| :--- | :---: | :---: |
| Treatment | Failure | Success |
| Disturbed | 32 | 22 |
| Undisturbed | 25 | 25 |

(a) Construct a 95 percent confidence interval for the difference in the population proportions of nest success. Assume that it is reasonable to regard these samples as representative of the corresponding populations.
(b) Does there appear to be a difference between the proportions of success? Provide statistical justification for your answer.

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## Inference for a Single Mean

A company manufactures portable music devices-called "mBoxes"-for the listening pleasure of on-the-go teens. The mBox uses batteries that are advertised to provide, on average, 25 hours of continuous use. The students in Mr. Jones's statistics class-looking for any excuse to listen to music while doing their homework-decide to use their statistics to test this advertising claim. To do this, new batteries are installed in eight randomly selected mBoxes, and they are used only in Mr. Jones's class until the batteries run down. Here are the results for the lifetimes of the batteries (in hours):

$$
\begin{array}{llllllll}
15 & 22 & 26 & 25 & 21 & 27 & 18 & 22
\end{array}
$$

(a) Assess graphically the plausibility of any necessary assumptions for your inference procedure.
(b) Do the data provide sufficient evidence that the claim by the battery manufacturer is not justified?

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## Inference for Two Independent Means

The perception of danger-i.e., teachers-is an important characteristic for survival of students in math classes. Students are often distracted from working on problems during class, and teachers will sometimes have to individually point out to students that they need to get to work. Usually students will resume working as the teacher approaches their desks. Are boys and girls equally aware of the impending presence of teachers? To help answer this question, a teacher randomly approached students who were talking about nonmath topics during class and observed their behavior. The outcome measure was the distance of the teacher from the student when he or she resumed the assigned tasks. (Only the selected student was counted if two students were off-task.) The teacher believes that boys are less sensitive to the presence of the teacher, and the teacher will have to be closer to boys than girls before they are sufficiently encouraged to resume their work. Data from this experiment appear in the table at right.

Approach Distance (m) to Elicit Work Resumption

| Boys | Girls |
| :---: | :---: |
| 3.19 | 2.09 |
| 2.34 | 1.96 |
| 2.45 | 1.85 |
| 2.71 | 2.45 |
| 1.90 | 2.77 |
| 2.12 | 2.55 |
| 2.56 | 2.44 |
| 3.41 | 2.80 |
| 2.41 | 3.27 |
| 2.66 | 2.01 |
| 2.86 | 3.49 |
| 2.44 | 2.75 |

(a) Using a graphical display of your choosing, assess the assumption that the distributions of approach distances are approximately normal.
State your conclusion in a few sentences.

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(b) Assuming that it is okay to proceed with a two-sample $t$ procedure, determine if there is sufficient evidence to conclude that there is a difference in the mean approach distances for the boys and girls.
(c) In a few sentences, state any concerns you have about your conclusions in part (b), based on your results from part (a). If you have no concerns, write "No concerns."

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## Inference for Matched Pairs

The diagram at right represents the human knee before a skiing accident. Note the posterior cruciate ligament. An ACL injury, the "scourge of skiers," can result in the displacement of the tibia when the ACL is twisted, angulated, or hyperextended.

The data presented in the table below are from two different measurements of displacement of the tibia relative to the femur in 16 patients suffering from ACL.

Investigators are concerned that anthropometric (i.e., with calipers) measurements, while less expensive than X-rays,
 may give biased results. Your task will be to address this question.
(a) Using the graphical display(s) of your choice, show that the assumptions necessary for the paired $t$-test are plausible.

| Anthro- <br> Metric <br> $(\mathbf{m m})$ | Radio- <br> Graphic <br> $(\mathbf{m m})$ |
| :---: | :---: |
| 13.0 | 12.5 |
| 17.0 | 16.5 |
| 10.5 | 9.5 |
| 8.0 | 9.0 |
| 12.5 | 11.5 |
| 18.0 | 16.5 |
| 14.0 | 15.5 |
| 10.0 | 7.5 |
| 10.0 | 7.5 |
| 11.0 | 14.5 |
| 10.0 | 6.5 |
| 8.5 | 5.5 |
| 8.0 | 12.5 |
| 12.5 | 8.5 |
| 11.5 | 16.5 |
| 16.0 | 8.5 |

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(b) Test the hypothesis that there is no difference between the anthropometric and radiographic measures of displacement. For purposes of the statistics, you may assume that these knees are a plausibly random sample of human knees.
(c) Write a short paragraph based on your analysis above, explaining your results for medical technicians. You should specifically advise them whether or not it is necessary to say in their reports which measure of displacement was used-this would be important if the measures give different results.

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## Inference with Chi-Square

In late May and early June on beaches on the eastern coasts of North America, it is common to see male horseshoe crabs overturned by the waves approaching the beach. Such "stranded" males must right themselves or face death. Researchers are interested in determining if, in the ebb and flow of the surf, older males tend to be stranded more than younger males. The table below includes data on horseshoe crabs brought in during a single tide; the "nonstranded" males include those attached to nesting females and unattached males found crowding around the nesting couples.

Age vs. Stranding in Crabs

| Stranding | Young | Intermediate | Old |
| :---: | :---: | :---: | :---: |
| Stranded | 41 | 125 | 52 |
| Not stranded | 153 | 364 | 70 |

Do the data support the contention that there is an association between stranding and age of the horseshoe crab? Provide statistical justification for your response.

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## Inference with Regression

The computer output given at the right shows an analysis of 31 ancient Roman coins. The investigators were interested in the metallic content of the coins as a method for identifying the mint location. Each data point represents the percent by weight of the coin that is gold versus the percent by weight of the coin that is tin.
(a) What is the least squares line for estimating the percent by weight of gold?

Percent by Weight Gold vs. Percent by Weight Tin


Linear Fit
Gold $=0.1682722+0.621629$ Tin

Summary of Fit

| RSquare | 0.117 |
| :--- | :--- |
| RSquare Adj | 0.086 |
| Root Mean Square Error | 0.084 |
| Observations | $\mathrm{N}=31$ |

Analysis of Variance

| Source | DF | SS | MS | F Ratio |
| :--- | :--- | :--- | :--- | :--- |
| Model | 1 | 0.0273 | 0.027 | 3.8339 |
| Error | 29 | 0.206 | 0.007 | Prob $>$ F |
| Total | 30 | 0.234 |  | 0.0599 |

Parameter Estimates

| Term | Estimate | Std <br> Error | $t$ <br> Ratio | Prob |
| :--- | ---: | ---: | ---: | ---: |
| Intercept | 0.168 | 0.0669 | 2.52 | 0.0176 |
| Tin | 0.6217 | 0.3175 | 1.96 | 0.0599 |

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(c) Based on your responses in parts (a) and (b), does it appear that there is a linear relationship between the percent by weight of gold and the percent by weight of tin for ancient Roman coins? Provide statistical justification for your response.

