AP ${ }^{\otimes}$ Statistics
Multiple-Choice Answer Key

| No. | Correct Answer |
| :---: | :---: |
| 1 | B |
| 2 | B |
| 3 | E |
| 4 | B |
| 5 | C |
| 6 | E |
| 7 | E |
| 8 | C |
| 9 | A |
| 10 | A |
| 11 | C |
| 12 | A |
| 13 | C |
| 14 | D |
| 15 | D |
| 16 | D |
| 17 | D |
| 18 | C |
| 19 | D |
| 20 | D |
| 21 | D |
| 22 | D |
| 23 | B |
| 24 | E |
| 25 | B |
| 26 | C |
| 27 | A |
| 28 | A |
| 29 | B |
| 30 | E |


| No.Correct <br> Answer |  |
| :---: | :---: |
| 31 | A |
| 32 | C |
| 33 | A |
| 34 | B |
| 35 | C |
| 36 | D |
| 37 | E |
| 38 | A |
| 39 | D |
| 40 | C |

# $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 1

## Intent of Question:

The primary goals of this question are to assess the student's ability to: (a) graphically display a small data set in a meaningful way; (b) describe the distribution of the data; and (c) evaluate how the addition of an outlier affects the measures of central tendency of the data.

## Sample Correct Response:

a. Two different graphical displays that would be possible correct responses are shown below.

b. The median amount of caffeine in popular beverages is 50 mg . The IQR of caffeine is 39.5 mg . (Note: The standard deviation of caffeine is 41.37 mg , but it is not expected that the student will obtain this statistic directly from the graph.) The distribution for the amount of caffeine in beverages is skewed right with an outlier at 165 mg .
c. The mean would increase for the larger data set, since it is affected by outliers. However, the median would not substantially change since it is resistant to outliers. (Note: The median may shift ever so slightly, since the number of values in the data set changes.)

## Scoring:

Parts (a), (b), and (c) are each scored as essentially correct, partially correct, or incorrect.

## Part (a):

- is essentially correct if the student presents a correct graphical display such as a boxplot or histogram, with complete labeling of that display.
- is partially correct if the student displays the data correctly but does not label the graph completely $O R$ labels the graph completely but makes one or two minor errors on a graphical display that is otherwise correct.
- is incorrect if the student uses an incorrect graphical display (or no graphical display), with or without correct labels.


# $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 1 (continued)

## Part (b):

- is essentially correct if the student correctly addresses the center, shape, and spread of the data in context.
- is partially correct if the student correctly addresses two of the three attributes (center, shape, and spread) of the data.
- is incorrect if the student addresses fewer than two of the three attributes.


## Part (c):

- is essentially correct if the student presents correct comparisons between the original (i.e., without the 300 data point included) and new (i.e., with the 300 data point included) sets for both the mean and median.
- is partially correct if the student presents a correct comparison between the original (i.e., without the 300 data point included) and new (i.e., with the 300 data point included) sets for the mean $O R$ the median.
- is incorrect if the student does not present a correct comparison or presents no comparison.


## 4 Complete Response

All three parts are essentially correct.

## 3 Substantial Response

Two parts are essentially correct, and one part is partially correct.

## 2 Developing Response

Two parts are essentially correct, and no part is partially correct.
OR
One part is essentially correct, and two parts are partially correct.
OR
Three parts are partially correct.

## 1 Minimal Response

One part is essentially correct, and either one part or no part is partially correct.
OR
No parts are essentially correct, and two parts are partially correct.

## 0 <br> No Credit

Response does not meet criteria for at least the minimal level.

# $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 2

## Intent of Question:

This question assesses the student's ability to: (a) identify the type of design and response variable for a statistical study; (b) propose a better design for the study than the one presented; and (c) identify an appropriate statistical test to use in the analysis of the data.

## Sample Correct Response:

a. This is a completely randomized design. The response variable is the amount of tread wear, which can be found by taking the difference in the tread on the rear tire at the outset and the tread on the rear tire after six months of use.
b. Place one tire with the new compound and one tire with the standard compound on each bicycle. Randomly choose 30 of the 60 bicycles, and place the tire with the new compound on the front for those bicycles and the tire with the new compound on the rear for the remaining 30 bicycles. Randomly distribute the bicycles to the students. Since students use bicycles differently, this design will ensure that each tire is exposed to similar conditions. (Note: This design would be stronger if the front and rear positions of the new and standard tires on each bicycle are interchanged after the first three months of the study.)
c. A matched-pairs $t$-test would determine whether tires produced using the new compound have longer tread life than tires produced using the standard compound.

## Scoring:

Parts (a) and (b) are each scored as essentially correct, partially correct, or incorrect. Part (c) is scored as essentially correct or incorrect.

## Part (a):

- is essentially correct if the student identifies both the type of design and the response variable correctly.
- is partially correct if the student identifies either the type of design or the response variable correctly.
- is incorrect if the student identifies neither the type of design nor the response variable correctly.


## Part (b):

- is essentially correct if the student suggests a better design than the one proposed by the researcher in part (a) and describes it sufficiently.
- is partially correct if the student suggests a better design with a less-than-sufficient explanation.
- is incorrect if the student does not suggest a better design.


## Part (c):

- is essentially correct if the student identifies a correct test.
- is incorrect if the student does not identify a correct test.


# AP ${ }^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 2 (continued)

4 Complete Response
All three parts are essentially correct.
3 Substantial Response
Two parts are essentially correct, and one part is partially correct.
2 Developing Response
Two parts are essentially correct, and no part is partially correct.
OR
One part is essentially correct, and two parts are partially correct.
1 Minimal Response
One part is essentially correct, and either one part or no part is partially correct.
OR
No parts are essentially correct, and two parts are partially correct.
0 No Credit
Response does not meet criteria for at least the minimal level.

# $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 3

## Intent of Question:

This question requires the student to apply knowledge of a binomial distribution in the computation of probabilities.

## Sample Correct Response:

a. The veterinarian will correctly conclude that the H6N2 virus is not present in the flock if no more than two chickens have false positive results. The probability that this occurs is

$$
\binom{10}{0}(0.05)^{0}(0.95)^{10}+\binom{10}{1}(0.05)^{1}(0.95)^{9}+\binom{10}{2}(0.05)^{2}(0.95)^{8}=0.9884964
$$

b. Using the answer from part (a), the probability that the veterinarian will incorrectly conclude that the H6N2 virus is present in the flock is

$$
1-0.9884964=0.0115036
$$

c. The veterinarian will correctly conclude that the H 6 N 2 virus is present in the flock if at least three chickens show positive results. The probability that this occurs is

$$
\begin{aligned}
& \sum_{\mathrm{k}=3}^{10}\binom{10}{\mathrm{k}}(0.90)^{\mathrm{k}}(0.10)^{10-\mathrm{k}}=1-\left[\binom{10}{0}(0.90)^{0}(0.10)^{10}+\binom{10}{1}(0.90)^{1}(0.10)^{9}+\binom{10}{2}(0.90)^{2}(0.10)^{8}\right] \\
& =0.9999996
\end{aligned}
$$

d. The veterinarian will correctly conclude that the H 6 N 2 virus is present in the flock if at least three chickens show positive results. The probability that a randomly selected chicken shows a positive result is

$$
\mathrm{P}(\text { true positive }) \times \mathrm{P}(\text { infected chicken })+\mathrm{P}(\text { false positive }) \times \mathrm{P}(\text { non-infected chicken })
$$

$$
=(0.90)(0.20)+(0.05)(0.80)=0.22
$$

Then the probability that at least three chickens show positive results is

$$
\begin{aligned}
& \sum_{\mathrm{k}=3}^{10}\binom{10}{\mathrm{k}}(0.22)^{\mathrm{k}}(0.78)^{10-\mathrm{k}}=1-\left[\binom{10}{0}(0.22)^{0}(0.78)^{10}+\binom{10}{1}(0.22)^{1}(0.78)^{9}+\binom{10}{2}(0.22)^{2}(0.78)^{8}\right] \\
& =0.3831197
\end{aligned}
$$

## Scoring:

Parts (a), (b), (c), and (d) are each scored as essentially correct, partially correct, or incorrect.

# $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 3 (continued)

## Parts (a) through (d):

- are each essentially correct if the student computes a correct probability and fully supports the probability with correct work. Although the numerical results of intermediate computations are not required in the supporting work, students will need to show enough evidence of the correct process and numbers utilized in that process to arrive at the final numerical probability value in each part.
- are each partially correct if the student computes a correct probability but the supporting work is incomplete (there must be some correct supporting work to earn partial credit), $O R$ the computed probability is incorrect but the supporting work is complete and a correct process is presented. In the latter instance, the error in computed probability should be the result of a computation mistake or minor transcription error only.
- are each incorrect if the student does not compute a correct probability and does not present a sufficient amount of supporting work.

Each essentially correct response in each part is worth 1 point, each partially correct response is worth $1 / 2$ point, and each incorrect response is worth 0 points.

## 4 Complete Response

A total of 4 points are earned.

## 3 Substantial Response

A total of 3 points are earned.

## 2 Developing Response

A total of 2 points are earned.
1 Minimal Response
A total of 1 point is earned.

## 0 No Credit

A total of 0 points are earned.
Note: If the total score earned for the four parts is not a whole number of points (i.e., $3^{1 / 2}$ points, $2^{1 / 2}$ points, $11 / 2$ points, or $1 / 2$ point), then the quality of the overall work in all four parts collectively needs to be evaluated. If the overall quality warrants, the total score should be moved up $1 / 2$ point to the next higher whole number score; if that is unwarranted, then the total score should be moved down $1 / 2$ point to the next lower whole number score. The general student performance expectations found in the AP Statistics Course Description are helpful in making this judgment. In the May 2007, May 2008 version of that publication, the performance expectations can be found on pages 27-28.

## $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines

## Question 4

## Intent of Question:

This question assesses the student's ability to identify and conduct the appropriate statistical test to investigate a claim and to use the results to answer a follow-up question.

## Sample Correct Response:

a. To test the statisticians' claim, a chi-square goodness of fit test should be performed.

Null hypothesis: Population proportions for the past summer are consistent with the proportions that are being used by the city for planning purposes and for more efficiently targeting the introduction of future programs.

Alternative hypothesis: Population proportions for the past summer are not consistent with the proportions that are being used by the city for planning purposes and for more efficiently targeting the introduction of future programs.

Check of conditions:

- Data were collected from a simple random sample.
- Expected cell counts are each greater than 5. (See column three of the table below.)

| District | Observed, $O$ | Expected, $E$ | $\frac{(O-E)^{2}}{E}$ |
| :---: | :---: | :---: | :---: |
| A | 100 | 76.8 | 7.01 |
| B | 35 | 28.8 | 1.33 |
| C | 40 | 24 | 10.67 |
| D | 22 | 64.8 | 28.27 |
| E | 12 | 12 | 0.00 |
| F | 31 | 33.6 | 0.20 |
|  | Sum |  | $\chi^{2}=47.48$ |

$p$-value $<0.001$, with 5 degrees of freedom
Since the $p$-value is so small, we have convincing evidence that the population proportions for the past summer are not consistent with the proportions that are being used by the city.
b. Based on the chi-square table, it appears that District D showed the greatest change in participation since 2000, because it contributed the greatest amount toward the calculation of the chi-square value.

# $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 4 (continued)

## Scoring:

The part (a) score is comprised of three separate components-stating the null and alternative hypotheses and verifying the conditions for inference; carrying out the mechanics of the test; and stating a correct conclusion in the context of the question. Each component is scored as essentially correct or incorrect. Part (b) of the question is comprised of a single component that is scored as essentially correct or incorrect.

## Part (a)—Hypotheses/Conditions:

- is essentially correct if the student states the correct null and alternative hypotheses and verifies the conditions for inference for this test.
- is incorrect if the student does not present correct information for both the hypotheses to be tested and a verification of the conditions for inference.


## Part (a)—Mechanics:

- is essentially correct if the student carries out the mechanics of the test. There must be a sufficient level of detail in the supporting computations in order to earn credit for this component.
- is incorrect if the student does not display correct mechanics for the test.


## Part (a)-Conclusion:

- is essentially correct if the student presents a correct conclusion in the context of the question linked to the $p$-value.
- is incorrect if the student is unable to present a correct conclusion, or no conclusion is presented.


## Part (b):

- is essentially correct if the student identifies the correct district and supports the choice of that district.
- is incorrect if the student identifies an incorrect district or does not adequately support the choice of the correct district.


## 4 Complete Response

Four components are essentially correct.

## 3 Substantial Response

Three components are essentially correct.

## 2 Developing Response

Two components are essentially correct.

## 1 Minimal Response

One component is essentially correct
0 No Credit
No components are essentially correct.

# AP ${ }^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 5

## Intent of Question:

The purpose of this question is to assess the student's ability to determine whether a cause-and-effect conclusion is justified in a particular situation and to make appropriate conclusions using available data.

## Sample Correct Response:

a. Since this was an observational study, a cause-and-effect conclusion is not justified. Sedentary women could differ from walkers with respect to other factors that affect BMD. For example, walkers may be more likely to avoid smoking and may be more careful about including sources of calcium in their diets. Simply encouraging sedentary women to walk more may not achieve the improvement in average BMD level shown in this study, if other differences between the two groups are ignored.
b. The plots of the data and the estimated regression lines suggest that increased calcium intake from milk consumption by women in their 20s is associated with higher average BMD levels. (Note: Higher levels of milk consumption by women in their 20s could be associated with a history of higher levels of milk consumption as children and teenagers and later as adults.)
c. This limits the conclusions that can be made about possible effects of exercise on BMD. If higher levels of milk consumption provide calcium needed to promote higher BMD levels, and sedentary women tend to have lower levels of milk consumption than walkers do, then all or part of the difference in BMD means for the two groups could be due to higher levels of calcium intake among walkers and not the physical activity.

## Scoring:

Parts (a), (b), and (c) are each scored as essentially correct, partially correct, or incorrect.

## Part (a):

- is essentially correct if the student clearly identifies another factor (besides walking) that could be contributing to the difference in BMD levels between the two groups OR explains the observational nature of the study.
- is partially correct if the student correctly realizes that there are other contributing factors but does not address at least one of them clearly in the response.
- is incorrect if the student's response does not meet at least the criteria described in the partially correct category.


## Part (b):

- is essentially correct if the student presents a correct statement of the relationship between calcium intake from milk consumption and BMD levels.
- is partially correct if the student's response suggests the correct relationship between calcium intake and BMD levels, but the description of the relationship is incomplete or is not clearly stated.
- is incorrect if the student's response does not meet at least the criteria described in the partially correct category.


# $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 5 (continued)

## Part (c):

- is essentially correct if the student uses the information in part (b) in an explanation that identifies the limitations associated with the conclusion in part (a).
- is partially correct if the student makes correct observations regarding the limitations of the conclusion in part (a) but does not link those observations strongly enough to the information presented in part (b).
- is incorrect if the student's response does not meet at least the criteria described in the partially correct category.


## 4 Complete Response

All three parts are essentially correct.

## 3 Substantial Response

Two parts are essentially correct, and one part is partially correct.
2 Developing Response
Two parts are essentially correct, and no part is partially correct.
OR
One part is essentially correct, and two parts are partially correct.
OR
Three parts are partially correct.

## 1 Minimal Response

One part is essentially correct, and either one part or no part is partially correct.
OR
No parts are essentially correct, and two parts are partially correct.
0 No Credit
Response does not meet criteria for at least the minimal level.

# $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 6

## Intent of Question:

The purpose of this question is to assess the student's ability to work with a linear regression model; describe how a simulation could be performed in a study; and use that simulation method to determine which of two possible expressions provides an appropriate method for constructing a confidence interval for the slope of a least-squares regression line with an intercept of 0 .

## Sample Correct Response:

a. One assumption is that the random errors at each volume have the same variance. The level of variation in the mass measurements about the true mass does not vary with the volume. A second assumption is that the random errors at each volume are normally distributed.
b. A 95 percent confidence interval for the slope is $1.3812 \pm(2.365)(0.00876)$ or $(1.3605,1.4019) \mathrm{kg} / \mathrm{mL}$. Since a 95 percent confidence interval has a probability of 0.95 of containing the true value, and the interval provided by the data includes 1.37 , this result is consistent with a material with a true density of $1.37 \mathrm{~kg} / \mathrm{mL}$.
c. The intercept represents the mean mass in kilograms for a sample with a volume of 0 .
d. Since a sample with no volume should have no mass, it would be very reasonable to consider a regression model with an intercept of 0 .
e. The basic steps for simulating a data set with nine observations are:

- Use a calculator or computer to generate a random observation $z$ from a standard normal distribution.
- For a specific volume, compute a simulated mass measurement as $y=1.3966 \times$ volume $+0.7925 \times z$.
- Repeat each step in the previous two bulleted statements for each of the nine amounts- $10,20,30,40,50$, $60,70,80$, and 90 - to obtain a sample of nine mass measurements.
f. The engineer would need to simulate a large number of data sets, say 10,000 , using the procedure described in part (e). For each simulated data set, the engineer would need to fit the least-squares regression line with an intercept of 0 to obtain the estimated slope and its standard error and evaluate both of the proposed formulas for constructing a 95 percent confidence interval for the slope. For the appropriate method, about 95 percent of the simulated confidence intervals should contain 1.3966, the slope of the "true" line used to simulate the data sets.


## Scoring:

There are six parts for this question. Parts (a) and (b) are scored together as a single score-essentially correct, partially correct, or incorrect. Parts (c) and (d) are scored together as a single score-also as essentially correct, partially correct, or incorrect. Part (e) is scored as essentially correct, partially correct, or incorrect. Part (f) is scored as essentially correct, partially correct, or incorrect.

Each essentially correct response in each part is worth 1 point, each partially correct response is worth $1 / 2$ point, and each incorrect response is worth 0 points.

# $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 6 (continued)

## Parts (a) and (b):

- are essentially correct if the student presents two correct assumptions about the distribution of the random errors in part (a) and constructs a correct confidence interval in part (b) with a corresponding correct conclusion in context.
- are partially correct if the student correctly responds to part (a) in its entirety but only partially correctly to part (b), OR correctly responds to part (b) in its entirety but only partially correctly to part (a), OR responds partially correctly to each of the two parts. In order to earn credit for a partially correct response in either part, the part (a) response must have one correct assumption stated, and the part (b) response must have a correct confidence interval with an incorrect or incomplete conclusion $O R$ a confidence interval that contains only a computational error with a corresponding conclusion that is correct based on that interval.
- are incorrect if the student's response does not meet at least the criteria described in the partially correct category.


## Parts (c) and (d):

- are essentially correct if the student presents a correct interpretation for the intercept in part (c) and a correct explanation in part (d) that supports the use of an intercept of 0 for the alternative model.
- are partially correct if the student answers either part (c) or part (d) correctly.
- are incorrect if the student's response does not meet at least the criteria described in the partially correct category.


## Part (e)

- is essentially correct if the student presents a correct method for simulating a data set with nine observations.
- is partially correct if the student presents an incomplete method for simulating the data set. The method must convey some information beyond the general level (i.e., use a random number table or use the random number generator on the calculator).
- is incorrect if the student's response does not meet at least the criteria described in the partially correct category.


## Part (f)

- is essentially correct if the student fully explains the process for determining which of the two provided expressions will result in a confidence interval for the slope of a least-squares regression line for the model with an intercept of 0 .
- is partially correct if the student utilizes the information in part (e) to fit the least-squares regression line with an intercept of 0 for the purpose of obtaining an estimated slope and standard error but does not complete the process $O R$ makes a mistake in carrying out this process.
- is incorrect if the student's response does not meet at least the criteria described in the partially correct category.


# $A P^{\circledR}$ Statistics <br> Free-Response Scoring Guidelines 

## Question 6 (continued)

## 4 Complete Response

A total of 4 points are earned.

## 3 Substantial Response

A total of 3 points are earned.

## 2 Developing Response

A total of 2 points are earned.

## 1 Minimal Response

A total of 1 point is earned.

## 0 No Credit

A total of 0 points are earned.
Note: If the total score earned for the four parts is not a whole number of points (i.e., $3^{1 / 2}$ points, $2^{1 / 2}$ points, $1^{11 / 2}$ points, or $1 / 2$ point), then the quality of the overall work in all four parts collectively needs to be evaluated. If the overall quality warrants, the total score should be moved up $1 / 2$ point to the next higher whole number score; if that is unwarranted, then the total score should be moved down $1 / 2$ point to the next lower whole number score. The general student performance expectations found in the AP Statistics Course Description are helpful in making this judgment. In the May 2007, May 2008 version of that publication, the performance expectations can be found on pages 27-28.

